

5.4 Stress Sequence Development

In order to predict the crack-growth behavior of an aircraft structure, the designer needs to know the sequence of stress cycles applied during the life of the structure. This stress history for a new design is developed from the service life requirements and the mission profile information specified by the procurement activity. Based on this information a repeated load history due to ground handling, flight maneuvers, gusts, pressurization, landing, store ejection, and any other load source is developed. The stress history at any given point is then determined from the applicable load/stress relations. Giessler, et al. [1981] describes this procedure in detail.

This section will outline the necessary steps and illustrate the development of a simplified stress sequence for the purpose of showing the effect of various sequence characteristics on crack-growth behavior. Understanding of these effects is of great importance in determining the damage tolerance of a structure.

5.4.1 Service Life Description and Mission Profiles

The load sequence developer works from the service life requirement summary and the mission profiles as given by the aircraft procurement documents. The service life data contains the total flight hours, expected calendar year life, number of missions to be flown, identification of mission types, and number of touch and go and full stop landings. The mission profile description provides the time variation of the airspeed, altitude, and gross weight such as illustrated in [Figure 5.4.1](#). Each mission is divided into segments, as shown, which can be easily characterized by the type and frequency of the various load sources.

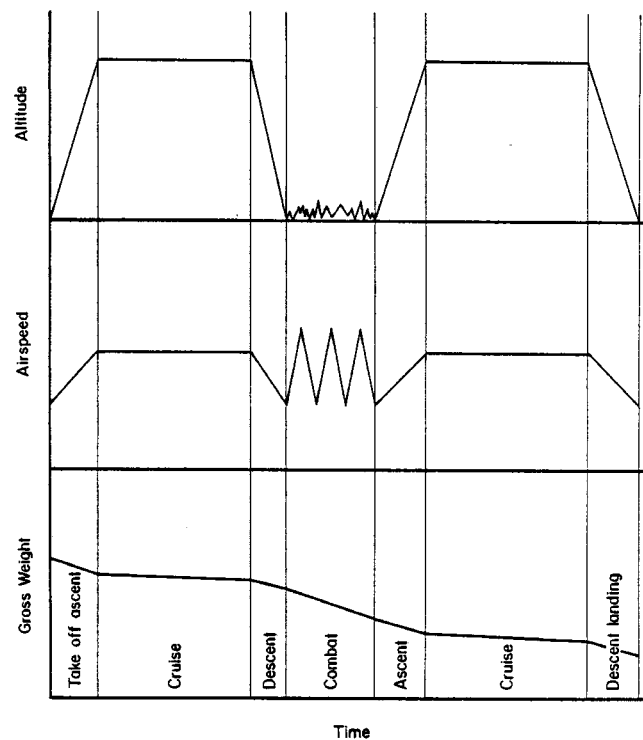


Figure 5.4.1. Mission Profile and Mission Segments

The load spectrum for each mission segment is characterized by a table of occurrences of a load parameter. The commonly used parameter is the normal load factor at the aircraft center-of-gravity, n_z . Such a table can be presented as an exceedance plot, which shows the number of occurrences that exceed specified values during a specified time period.

MIL-A-8866 presents tabular exceedance data for six classes of aircraft, broken out by mission segment. The number of identified segments varies from three to seven. These tables give the number of exceedances per 1,000 mission hours. The total number of exceedances is on the order of $10^5 - 5 \times 10^5$. [Figure 5.4.2](#) shows a plot of the composite maneuver spectrum for the six classes of aircraft. This composite was made by summing the exceedances of the mission segments for each class of aircraft.

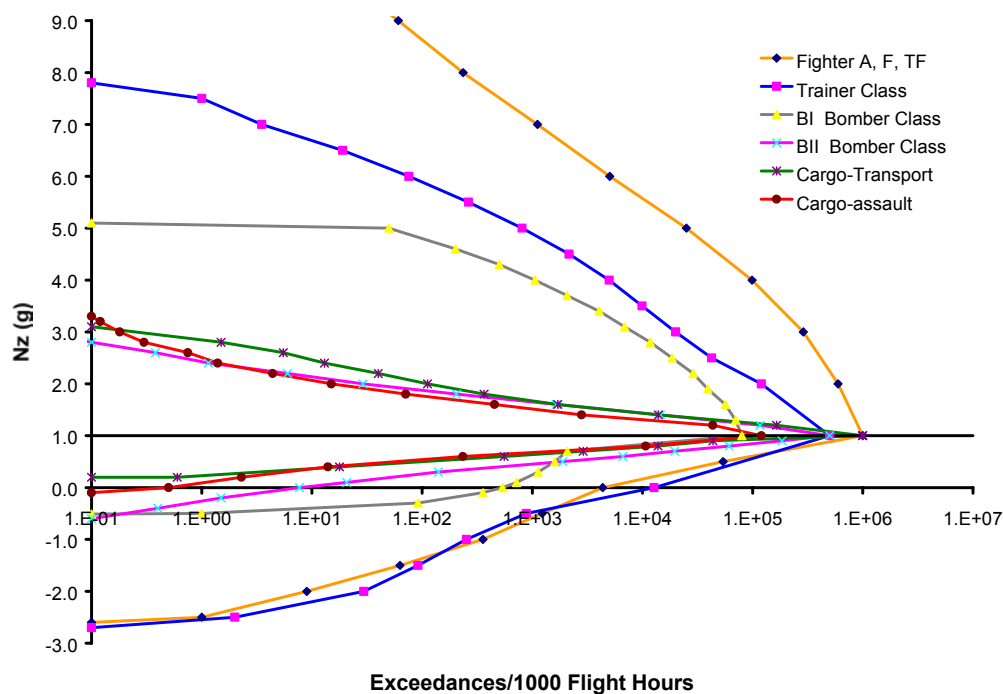


Figure 5.4.2. Maneuver Spectra According to MIL-A-8866

These three basic pieces of information, the service life summary, the mission profiles, and the load factor spectra are converted into the loads history and the stress history at critical locations on the aircraft. This procedure is briefly described in the next section.

5.4.2 Sequence Development Techniques

The preparation of a flight-by-flight load sequence is done in five essential steps:

1. Prepare the representative life history mission ordering.
2. Define mission segment flight conditions.
3. Determine the number of maneuver and gust load cycles at each load level in each mission segment.
4. Order the maneuver and gust load cycles within each mission segment.

5. Place load cycles from other sources within each mission segment.

Giessler, et al. [1981] presents detailed instructions to accomplish these steps; however, brief descriptions are presented here for completeness.

The establishment of the mission sequence for the life history of the aircraft is usually done in a deterministic manner and is based on past observations of similar type aircraft. It is reasonable to assign missions in blocks as flying assignments usually follow specific groupings of missions as various flying skills are being stressed. Missions occurring relatively few times are usually interspersed throughout the sequence either singly or in small numbers. It is recommended that the severest missions be somewhat evenly spaced throughout the sequence. It is common to treat the sequence as a repeating block. Each block would contain all mission types and represent some proportion of the total flight history. Blocks of five hundred or one thousand flight hours have been found to be convenient. For example: for a 6,000 hour aircraft life, a 500 hour block would be repeated twelve times to obtain the total sequence.

Each mission is divided into segments for ease in defining the loading cycles. This division is specified in the mission profile. The same ordering of mission segments is used each time a mission occurs. This division is useful in two ways: it facilitates the identification of load spectra and it provides for the definition of the flight condition parameters. The flight condition parameters are the values of airspeed, altitude, gross weight, configuration, and time used for each mission segment. These conditions are selected from the mission profile to give a set of representative loading conditions for each segment. These are combined with the load level indicator to compute the loads.

The determination of the number and severity of loads assigned to each mission segment is based on a spectrum of a load level indicator. For most applications this is the normal load factor, n_z . This spectrum is obtained from analysis of previous usage of similar aircraft in the case of a design specification, or from current usage of the aircraft being analyzed in the case of an update to the design analysis. Some of the concerns that need to be considered when applying this information will now be discussed.

The load information for an aircraft structure is usually in the form of an exceedance spectrum. The spectrum is an interpretation of in-flight measurements of center-of-gravity accelerations or stresses at a particular location. The interpretation consists of a counting procedure, which counts accelerations (or stresses) of a certain magnitude, or their variation (range). Information on the various counting procedures can be found in Schijve [1963] and VanDyk [1972].

Typical exceedance spectra are given in [Figure 5.4.3](#) for a transport wing, bomber wing, and fighter wing. The ordinate represents the normal load factor, n_z . The abscissa represents the number of times a level on the vertical axis is exceeded. For example, using the transport spectrum in [Figure 5.4.3](#), level A is exceeded n_1 times and level B is exceeded n_2 times. This means that there will be $n_1 - n_2$ events of a load between levels A and B . These loads will be lower than B , but higher than A . The exact magnitude of any one of the $n_1 - n_2$ loads remains undetermined.

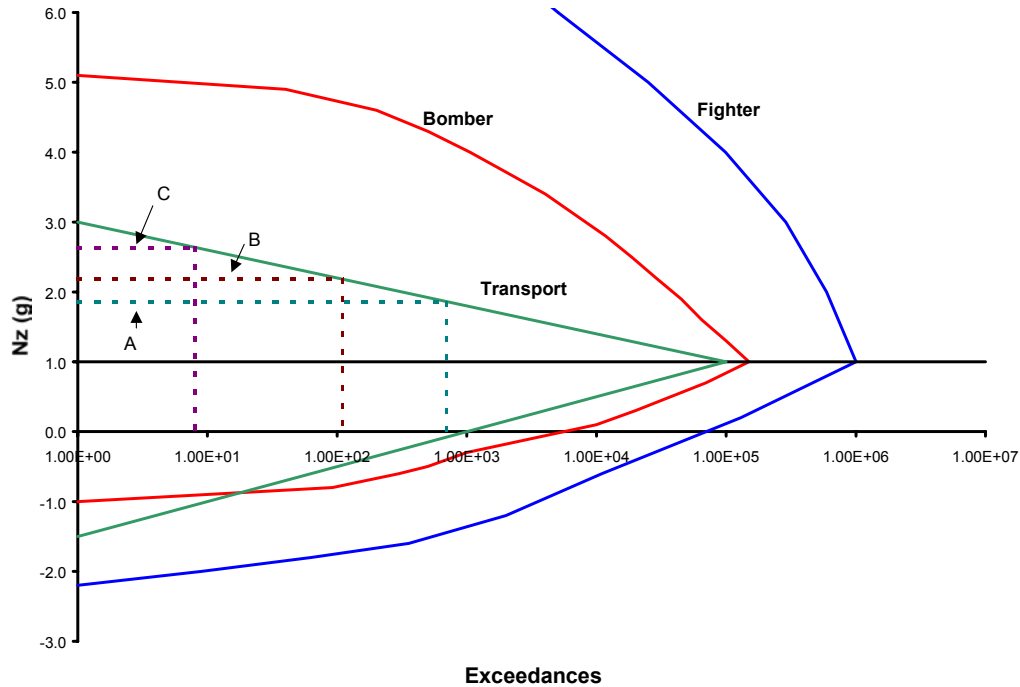


Figure 5.4.3. Exceedance Spectra for 1000 Hours

One can define an infinite number of load levels between A and B . However, there are only $n_1 - n_2$ occurrences, which means that while the number of load levels to be encountered is infinite; not every arbitrary load level will be experienced. Strictly speaking each of the $n_1 - n_2$ occurrences between A and B could be a different load level. If one chose to divide the distance between A and B into $n_1 - n_2$ equal parts, ΔA , each of these could occur once. Mathematically, a level $A + \Delta A$ will be exceeded $n_1 - 1$ times. Hence, there must be one occurrence between A and $A + \Delta A$. In practice, such small steps cannot be defined, nor is there a necessity for their definition.

If measurements were made again during an equal number of flight hours, the exceedance spectrum would be the same, but the actual load containment would be different. This means that the conversion of a spectrum into a stress history for crack-growth analysis will have to be arbitrary because one can only select one case out of unlimited possibilities.

Going to the top of the spectrum in [Figure 5.4.3](#), level C will be exceeded 10 times. There must be a level above C that is exceeded 9 times, one that is exceeded 8 times, etc. One could identify these levels, each of which would occur once. In view of the foregoing discussion this becomes extremely unrealistic. Imagine 10 levels above C at an equal spacing of ΔC , giving levels C , $C + \Delta C$, $C + 2\Delta C$, etc. If level C is exceeded 10 times, all of these exceedances may be of the level $C + 3\Delta C$ for another aircraft.

As a consequence, it is unrealistic to apply only one load of a certain level, which would imply that all loads in the history would have a different magnitude. Moreover, if high loads are beneficial for crack growth (retardation), it would be unconservative to apply once the level $C + \Delta C$, once $C + 2\Delta C$, etc., if some aircraft would only see 10 times C .

Hence, the maximum load level for a fatigue analysis should be selected at a reasonable number of exceedances. (This load level is called the clipping level). From crack-growth experiments

regarding the spectrum clipping level, it appears reasonable to select the highest level at 10 exceedances per 1,000 flights. This will be discussed in more detail in later sections. (Note that the maximum load used in the fatigue analysis has no relation whatsoever to the P_{xx} loads for residual strength analysis).

The same dilemma exists when lower load levels have to be selected. Obviously, the n loads in 1,000 hours will not be at n different levels. A number of discrete levels has to be selected. This requires a stepwise approximation of the spectrum, as in [Figure 5.4.4](#). As shown in the following table, the number of occurrences of each level follows easily from subtracting exceedances.

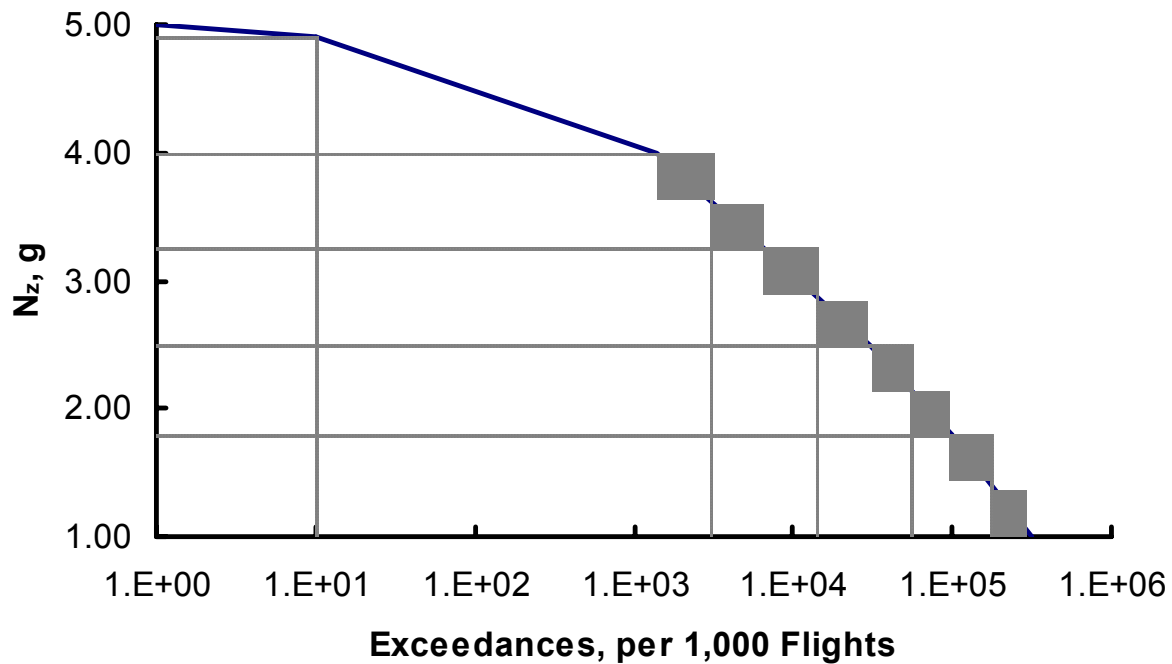


Figure 5.4.4. Stepped Approximation of Spectrum

Table 5.4.1. Occurrences Calculated from the Exceedances of [Figure 5.4.4](#)

Level	Exceedances	Occurrences
L_1	n_1	n_1
L_2	n_2	$n_2 - n_1$
L_3	n_3	$n_3 - n_2$
L_4	n_4	$n_4 - n_3$
L_5	n_5	$n_5 - n_4$

The more discrete load levels there are, the closer the stepwise approximation will approach the spectrum shape. On the other hand, the foregoing discussion shows that too many levels are unrealistic. The number of levels has to be chosen to give reliable crack-growth predictions.

Figure 5.4.5 shows results of crack-growth calculations in which the spectrum was approximated in different ways by selecting a different number of levels each time. If the stepped approximation is made too coarse (small number of levels) the resulting crack-growth curve differs largely from those obtained with finer approximations. However, if the number of levels is 8 or more, the crack-growth curves are identical for all practical purposes. A further refinement of the stepped approximation only increases the complexity of the calculation; it does not lead to a different (or better) crack-growth prediction. Crack-growth predictions contain many uncertainties anyway, which means that one would sacrifice efficiency to apparent sophistication by taking too many levels. It turns out that 8 to 10 positive levels (above the in-flight stationary load) are sufficient. The number of negative levels (below the in-flight stationary load) may be between 4 and 10.

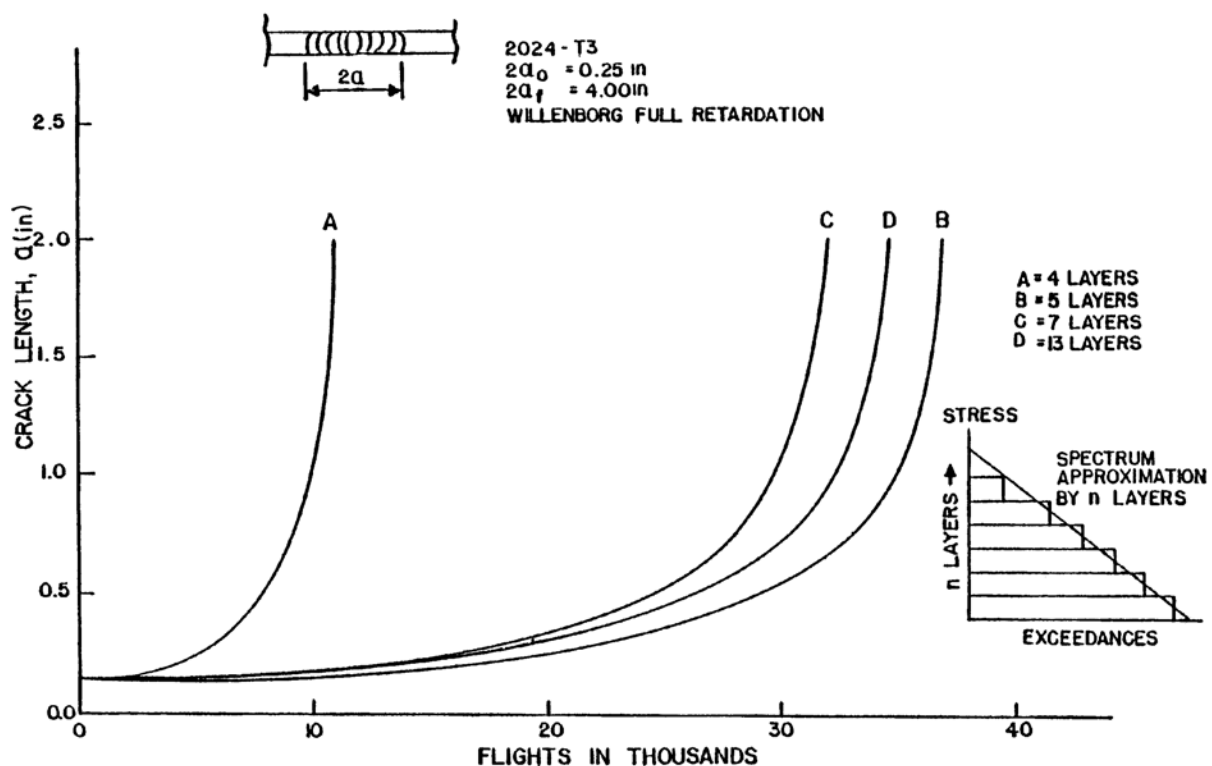


Figure 5.4.5. Fatigue-Crack Growth Behavior Under Various Spectra Approximations

Selection of the lowest positive level is also of importance, because it determines the total number of cycles in the crack-growth analysis. This level is called the truncation level. Within reasonable limits the lower truncation level has only a minor effect on the outcome of the crack-growth life. Therefore, it is recommended that this lower truncation level be selected on the basis of exceedances rather than on stresses. A number in the range of 10^5 - 5×10^5 exceedances per 1,000 flights seems reasonable. This will be discussed in more detail in Section 5.5.

EXAMPLE 5.4.1 Constructing Occurrences from Exceedance Information

This example illustrates how a stepped approximation can be constructed. Consider the positive load factor spectrum shown in [Figure 5.4.4](#).

First select the maximum level as the load which is exceeded 10 times in 1,000 flights. This is done by constructing a line from the 10 exceedance level to the curve and then constructing the horizontal to intersect the vertical axis. This gives $L_1 = 4.9$ g. Next construct a vertical from the 10^5 exceedance value. This line is extended until the area A_5 equals B_5 . The horizontal line defining the top of A_5 is extended to the vertical axis defining the level L_5 . In this case $L_5 = 1.8$ g.

Now the interval from L_5 to L_1 is divided into as many parts as desired. They may be equal or not. Current fighter aircraft practice uses 0.5 g intervals. After the vertical divisions are selected, horizontal lines are extended at L_2 , L_3 , and L_4 such that the enclosed areas (A_2, B_2), (A_3, B_3) and (A_4, B_4) are approximately equal. At that point the verticals are constructed to define n_2 , n_3 , and n_4 . This now gives the results:

Level	n_z	Exceedance	Occurrences
L_1	4.9	10	10
L_2	4.0	1400	1,390
L_3	3.25	6500	5,100
L_4	2.5	30,000	23,500
L_5	1.8	100,000	70,000
Total			100,000

This procedure is only used to construct the steps after the L_1 level. The L_1 level is taken as the intersection with the curve. It is seen that, as the exceedance plot tails off the high levels on n_z , to construct equal areas becomes difficult if not impossible. In the present example, in order to keep the exceedance value of 10 for the high level, L_1 could not extend beyond 5.0 g and the lower limit of the range could not go below 4.75 g. Now the range from 4.75 g to L_2 would need to be added to the number of levels. This would add high level occurrences that may not be realistic. It should be remembered that the exceedance plot is a curve faired through observed data and that the high level values are usually the result of very few observations.

This method of approximating the spectrum associates the level L_5 with the occurrence represented by a range extending on either side of L_5 and similarly for L_1 which was discussed above. An alternate procedure is to select the ranges first and then to associate the occurrences with the mid-point of the ranges.

After the levels and number of occurrences of the load indicator are determined for each mission segment, the actual loads are computed using the previously defined flight conditions and the specific load equations for the aircraft. Cycles are formed by combining the positive loads with the mean or negative loads. As there are more positive loads than negative loads, most cycles

are formed with the mean, or 1.0g steady flight condition, as the minimum value of the cycle. The assignment of the negative load cycles is usually on a random basis.

The sequencing of these load cycles is the next step. In order to achieve a realistic effect on the crack-growth analysis, care must be taken in establishing this sequence. Some guidelines are given below.

- a) Deterministic loads are placed directly in the sequence. Obviously, the ground load of the ground-air-ground (G-A-G) cycle will occur at the beginning and at the end of each flight. Similarly, maneuver loads associated with take-off will be at the beginning of the flight.
- b) Probabilistic loads due to gusts and maneuvers have to be arbitrarily assigned and sequenced. The assignment of the loads in a particular mission segment is made on a random basis to all flights containing that mission segment. This results in each flight of a particular mission having a different selection of loads. If a repeating block approach is used, then each flight in the block would be different. Sequencing of the assigned loads within a segment can be either random or deterministic. A deterministic low-high-low sequence has been shown by Schijve [1970, 1972] to be very similar to random loading for a gust spectrum. This sequence is also realistic for the combat maneuvering segments of fighter aircraft. Thus, the low-high-low sequence is recommended if programmed sequencing is considered rather than random sequencing.
- c) After determination of all the mission stresses, simplifications are sometimes possible. Usually the stresses will be given in tabular form. They will show an apparent variability. For example, if an acceleration, n_2 , is exceeded 10,000 times, this will not result in the exceedance of 10,000 times of a certain stress level, since n_2 causes a different stress in different missions or mission segments. However, if a stress exceedance spectrum is established for the various missions on the basis of the tabular stress history, it may turn out that two different missions may have nearly the same stress spectrum. In that case, the missions can be made equal for the purpose of crack-growth predictions.
- d) Placement of non-probabilistic load sources which occur a specified number of times in a flight is made on a deterministic basis. One such method is to place them after a certain number of occurrences of the probabilistic loads. This is reasonable for a random sequencing, however, if the sequencing has been low-high-low, then following the same method and placing these miscellaneous cycles in the proper location is suggested.

While the above discussions were primarily directed toward development of wing loadings, similar methods are used to obtain the load sequencing for other parts of the aircraft. Only the significant loading conditions will change.

5.4.3 Application of Simplified Stress Sequences for Design Studies

In the early design stage, not much is known about the anticipated stress histories. An exceedance spectrum based on previous experience is usually available. However, material selection may still have to be made, and operational stress levels may still have to be selected. Hence, it is impossible and premature to derive a detailed service life history as discussed in [Section 5.4.2](#). Yet, crack-growth calculations have to be made as part of the design trade-off studies. The designer wants to know the effect of design stress, structural geometry, and material

selection with respect to possible compliance with the damage-tolerance criteria, and with respect to aircraft weight and cost. Such studies can be made only if a reasonable service stress history is assumed. The following example shows how much a history can be derived in a simple way, if it is to be used only for comparative calculations.

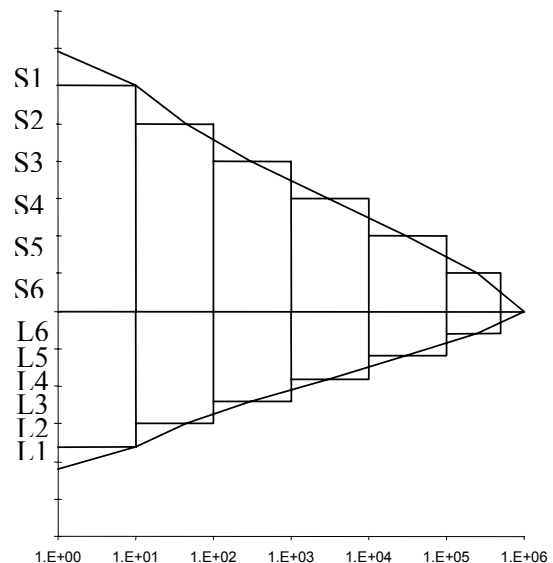
EXAMPLE 5.4.2 Construction of a Simple Stress Sequence

Consider the exceedance spectrum for 1,000 flights shown below. Instead of selecting stress levels for the discretization, it is much more efficient in this case to select exceedances. Since a large number of levels is not necessary in this stage, six levels were chosen in the example. The procedure would remain the same if more levels were to be selected.

The exceedances in the example were taken at 10 (in accordance with [Section 5.4.2](#)); 100; 1,000; 10,000; 100,000; and 500,000 (in accordance with [Section 5.4.2](#)). Vertical lines are drawn at these numbers, and the stepped approximation is made. This leads to the positive excursion levels, S_1 - S_6 , and the negative excursion levels, L_1 - L_6 , as shown below. The stress levels and exceedances are given in columns 1 and 2 of the table; subtraction gives the number of occurrences in column 3.

The highest stress level is likely to occur only once in the severest mission. Therefore, a mission *A* spectrum is selected, as shown in column 4, in which S_1 occurs once, and lower levels occur more frequently in accordance with the shape of the total spectrum. In order to use all 10 occurrences of level S_1 , it is necessary to have 10 missions *A* in 1,000 flights. The number of cycles used by 10 missions *A* is given in column 5. The occurrences from these missions are subtracted from the total number of occurrences (column 3) to give the occurrences in the remaining 990 flights (column 6).

The next severest mission is likely to have one cycle of level S_2 . Hence, the mission *B* spectrum in column 7 can be constructed in the same way as the mission *A* spectrum. Since 60 cycles of S_2 remain after mission *A*, mission *B* will occur 60 times in 1,000 flights. The 60 missions *B* will use the cycles shown in column 8, and the cycles remaining for the remaining 930 flights are given in column 9.



Composite			Mission A			Mission B		
<u>1</u> Level	<u>2</u> Exceedances	<u>3</u> Occurrences	<u>4</u> Occurr.	<u>5</u> 10 x	<u>6</u> Remain (= 3-5)	<u>7</u> Occurr.	<u>8</u> 60 x	<u>9</u> Remain (= 6-8)
S ₁	10	10	1	10	--	--	--	--
S ₂	100	90	3	30	60	1	60	--
S ₃	1,000	900	15	150	750	3	180	570
S ₄	10,000	9,000	48	480	8,520	17	1,020	7,500
S ₅	100,000	90,000	300	3,000	87,000	200	12,000	75,000
S ₆	500,000	400,000	1,900	19,000	381,000	1,500	90,000	291,000
Composite			Mission C			Mission D		
<u>1</u> Level			<u>10</u> Occurr.	<u>11</u> 570 x	<u>12</u> Remain (= 9-11)	<u>13</u> Occurr.	<u>14</u> 360 x	<u>15</u> Remain (= 12-14)
S ₁			--	--	--	--	--	--
S ₂			--	--	--	--	--	--
S ₃			1	570	--	--	--	--
S ₄			10	5,700	1,800	5	1,800	--
S ₅			100	57,000	18,000	50	18,000	--
S ₆			400	228,000	63,000	175	63,000	--

Level S_3 will occur once in a mission C , which is constructed in column 10. There remain 570 cycles S_3 , so there will be 570 missions C . These missions will use the cycles given in column 11, and the remaining cycles are given in column 12.

Mission	Number of Times	Repeat
D	6	Repeat 33 times
B	1	
C	19	
B	1	
D	6	
A	1	

There will be 10 missions A , 60 missions B , and 570 missions C in 1,000 flights, meaning that 360 flights remain. By dividing the remaining cycles in column 12 into 360 flights, a mission D spectrum is defined, as given in column 13. Consequently, all cycles have been accounted for.

A mission mix has to be constructed now. With mission *A* occurring 10 times per 1,000 flights, a 100-mission block could be selected. However, a smaller block would be more efficient. In the example, a 33-mission block can be conceived, as shown below. After 3 repetitions of this block (99 flights) one mission *A* is applied.

Mission	Number of Times	Repeat
D	6	Repeat 33 times
B	1	
C	19	
B	1	
D	6	
A	1	

The cycles in each mission are ordered in a low-high-low sequence. The negative excursion L_1 - L_6 are accounted for by combining them with the positive excursions of the same frequency of occurrence: L_1 forms a cycle with S_1 , L_2 with S_2 , etc.

To arrive at the stresses an approximate procedure has to be followed also. Given the flight duration, an acceleration spectrum (e.g., the 1,000 hours spectra given in MIL-A-8866B) can be converted approximately into a 1,000 flight spectrum. Limit load will usually be at a known value of n_z , e.g., 7.33g for a fighter or 2.5g for a transport. As a result, the vertical axis of the acceleration diagram can be converted into a scale that gives exceedances as a fraction of limit load. This is done in [Figure 5.4.6](#) for the MIL-A-8866B spectra of [Figure 5.4.2](#). A comparison of these figures will clarify the procedure.

Once the spectrum of the type of [Figure 5.4.6](#) is established, design trade-off studies are easy. Selecting different materials or different design stress levels S_1 - S_6 and L_1 - L_6 can be determined and the flight-by-flight spectrum is ready. Selection of a different design stress level results in a new set of S_1 - S_6 , and the calculations can be re-run.

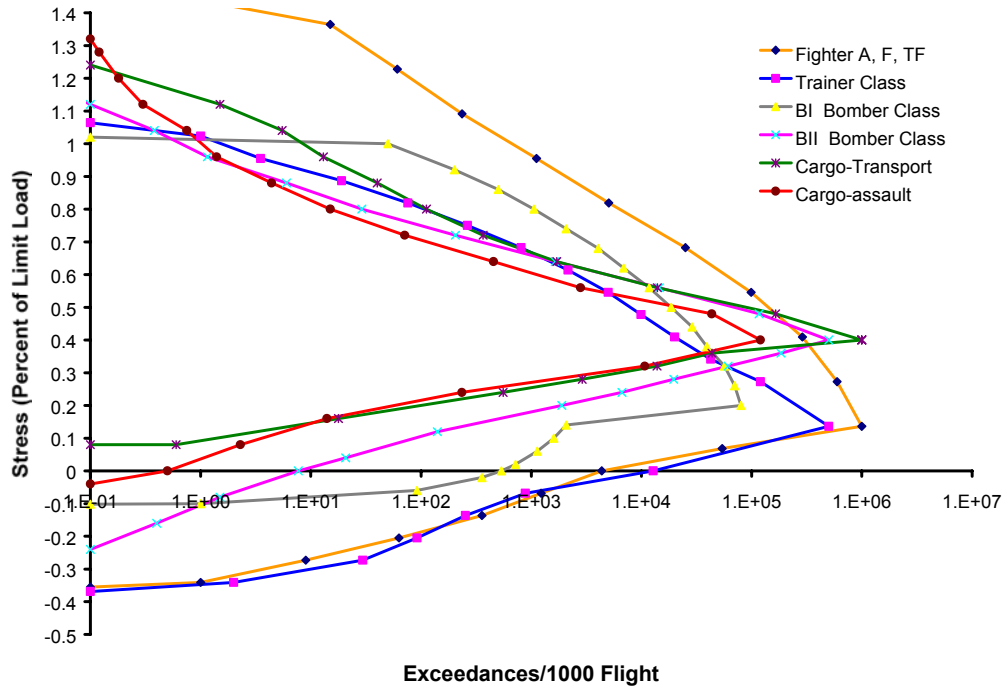


Figure 5.4.6. Approximate Stress Spectrum for 1000 Flights Based on MIL-A-8866B (USAF)

This shows the versatility of the spectrum derivation shown in [Example 5.4.2](#). It is a result of choosing exceedances to arrive at the stepped approximation of the spectrum, which means that the cycle content is always the same. If stress levels were selected instead, a change in spectrum shape or stress levels would always result in different cycle numbers. In that case, the whole procedure to arrive at the spectrum in [Example 5.4.2](#) would have to be repeated, and many more changes would have to be made to the computer program.

[Example 5.4.2](#) shows only a few levels. The spectrum could be approximated by more levels and more missions could be designed, but the same procedure can be used. In view of the comparative nature of the calculations in the early design stage, many more levels or missions are not really necessary.

Note: The stress history derived in this section is useful only for quick comparative calculations for trade-off studies.

The stress history developed in [Example 5.4.2](#) was applied to all the spectra from MIL-A-8866B (shown in [Figure 5.4.6](#)) to derive crack-growth curves. These results will be discussed in Section 5.5.3.